Final Review: Part 1

Monday, June 5, 2023 8:51 AM

preparation:

- practice problems (with solutions)
- practice midterms
- 2 new practice finals this week
- problem sets & webwork
- textbook

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series & taylor:

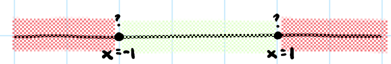
- decide if series converges or diverges ← battery of tests
  - if converges & need to compute limit ⟶ geometric series

$$\sum_{n=0}^{\infty} r^n = \begin{cases} \frac{1}{1-r} & \text{if } r<1 \\ \infty & \text{else} \end{cases}$$

ex) given taylor expansion of $\ln(1+x) \overset{a=0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots = \sum_{n\geq 1} (-1)^{n+1} \frac{x^n}{n}$

- determine for which x the above converges:

root test: $\lim_{n\to\infty} \sqrt[n]{a_n}$ $\quad a_n = \left|\frac{\overset{|-1 \cdot \alpha| = 1}{(-1)^{n+1}} x^n}{n}\right| = \frac{x^n}{n} \longrightarrow \lim_{n\to\infty} \sqrt[n]{\frac{x^n}{n}} = x \cdot \lim_{n\to\infty} \sqrt[n]{\frac{1}{n}} = x$ $\quad \left(\frac{1}{n}\right)^{1/n} \to 1$



- conclusion: root test says
  - converges if $|x| < 1$
  - diverges if $|x| > 1$

$|x| = 1 \to x = \pm 1$

$x=1$: $\sum_{n\geq 1} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots \xrightarrow{\frac{1}{n} \to 0}$ converges by alternating test

$x=-1$: $-\sum_{n\geq 1} \frac{1}{n} = -\underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\right)}_{\text{p-series w/ } p=1} \longrightarrow$ diverges by p-test (or integral test)

$\int_1^{\infty} \frac{1}{x} = \left[\ln(x)\right]_1^{\infty}$

taylor series: $a = 0$, then $f(x)$ differentiable

$$f(x) \overset{x\approx 0}{\approx} f(0) + \frac{f'(0)}{1!}(x-0)^1 + \frac{f''(0)}{2!} x^2 + \ldots$$
$$= \sum_{n\geq 0} \frac{f^{n}(0)}{n!} x^n$$

tricks: learn ...

memorize:
- $\cos(x)$, $\frac{1}{1-x}$
- $\sin(x)$
- e^x

1) if x substituted by $\blacksquare$ → plug $\blacksquare$ in taylor
2) $\int$ & derive
3) if polynomial → best approximation = itself

errors in taylor: if approximate @ order n ...

$$\text{error} = \left|\frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}\right| \quad \text{where } c \in (0, x)$$

ex) $\cos(x) \approx 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$ order 4 approximation $(a=0)$

$$\text{error}: \left|\frac{\cos^{(5)}(c)}{5!} x^5\right| \leq \frac{x^5}{5!} \overset{\text{if } x=0.1}{=} \frac{10^{-5}}{120}$$